Abstract Submitted
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The American Physical Society

Transversal motion and flow structure of fully nonlinear streaks in a laminar boundary layer JUAN ANGEL MARTIN, CARLOS MARTEL, Universidad Politecnica de Madrid, DENLIA TEAM — Typical streak computations present in the literature correspond to linear streaks or to small amplitude nonlinear streaks computed using DNS or nonlinear PSE. We use the Reduced Navier-Stokes (RNS) equations to compute the streamwise evolution of fully non-linear streaks with high amplitude in a laminar flat plate boundary layer. The RNS formulation provides Reynolds number independent solutions that are asymptotically exact in the limit $Re \gg 1$, it requires much less computational effort than DNS, and it does not have the consistency and convergence problems of the PSE. We present various streak computations to show that the flow configuration changes substantially when the amplitude of the streaks grows and the nonlinear effects come into play. The transversal motion (in the wall normal-streamwise plane) becomes more important and strongly distorts the streamwise velocity profiles, that end up being quite different from those of the linear case. We analyze in detail the resulting flow patterns for the nonlinearly saturated streaks and compare them with available experimental results.

Juan Angel Martin
Universidad Politecnica de Madrid

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Transversal motion and flow structure of fully nonlinear streaks in a laminar boundary layer

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November 17, 2011

Streaky flow in flat plate boundary layer

STREAKS

Streamwise long, spanwise thin structures, HS Down-LS Up.

Streaky flow in flat plate boundary layer

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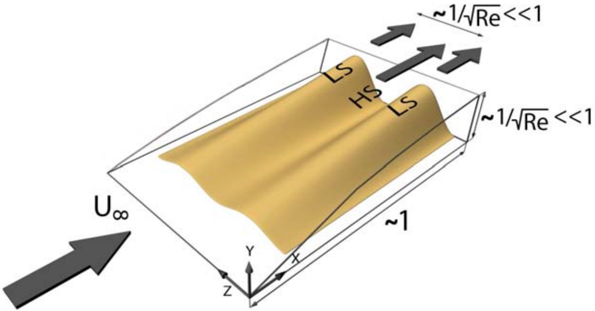
Streamwise long, spanwise thin structures, HS Down-LS Up.



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Transition to Turbulence delayed

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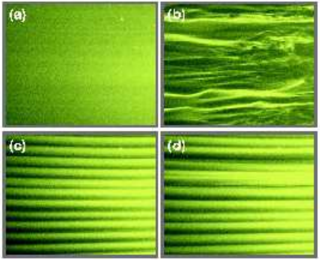
3D Streaky BL more stable to TS perturbations than 2D Blasius BL
Transition to Turbulence delayed, theoretically and experimentally

without TS **with TS**

STREAKS off

STREAKS on



Fransson & Talamelli & Brandt & Cossu PRL 2006

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3D Streaky BL more stable to TS perturbations than 2D Blasius BL
Transition to Turbulence delayed, theoretically and experimentally
Linear inviscid stability analysis found a critical streak amplitude

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Recent experiments produce stable streaks above critical amplitude

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Motivate computation of high amplitude streaks

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks

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Laminar nonlinear high intensity streaks
Tipically compute using DNS and nonlinear PSE

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- **DNS**
  Very CPU costly

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  Consistency problems

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$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks
Tipically compute using DNS and nonlinear PSE

- **DNS**
  Very CPU costly
- **PSE**
  Consistency problems

$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

Simplified boundary layer like formulation - **BRE's**

Streaks induced by Free Stream Perturbations.

Wundrow & Goldstein 1998

Leib & Wundrow & Goldstein 1999,2001

Ricco & Luo & Wu 2011

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks
Tipically compute using DNS and nonlinear PSE

- **DNS**
  Very CPU costly
- **PSE**
  Consistency problems

$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

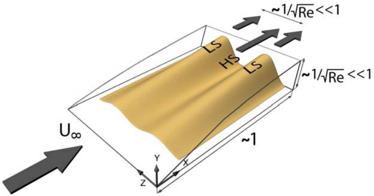
Simplified boundary layer like formulation - **BRE's**
Robust and fast computation
RNS compute the downstream evolution and flow pattern of high amplitude streaks

RNS 3D

$Re = U_\infty L/\nu \gg 1$

x scaled with L
y, z scaled with $\delta \sim L/\sqrt{Re}$
u scaled with $U\infty$
v, w scaled with $U_\infty/\sqrt{Re}$
p scaled with ρU_∞^2

2 short scales y, z + 1 long x



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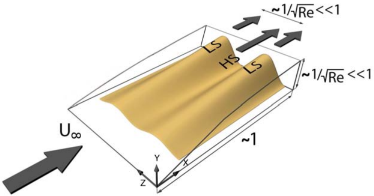

$$(u, v, w) = (u, v, w) + \frac{1}{Re}(u, v, w)_1 + \cdots$$

$$p = p + \frac{1}{Re}p_1 + \cdots$$

RNS 3D

$Re = U_\infty L/\nu \gg 1$

x scaled with L
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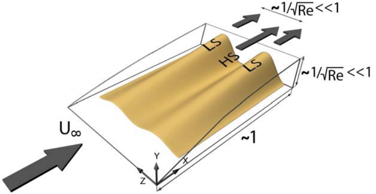


$$(u,v,w) = (u,v,w) + \frac{1}{Re}(u,v,w)_1 + \cdots$$

$$p = p + \frac{1}{Re}p_1 + \cdots$$

Standard 3D BL: 1 short scales y + 2 long x,z !!

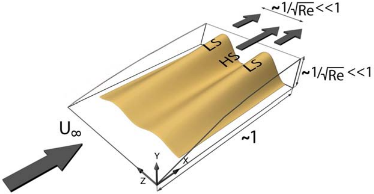
Schlichting BLT 1979

RNS 3D

At first order, M-y, M-z:

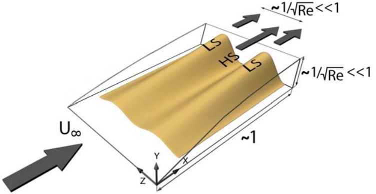
$$\left.\begin{array}{l} p_y = 0 \\ p_z = 0 \end{array}\right\} \Longrightarrow p = p_0(x)$$

Inviscid flow



RNS 3D

At first order, M-y, M-z:



$$\left.\begin{aligned} p_y &= 0 \\ p_z &= 0 \end{aligned}\right\} \Longrightarrow p = p_0(x)$$

Inviscid flow

And then, M-x, Cont, **M-y, M-z**

RNS 3D eqs.

$$u_x + v_y + w_z = 0$$

$$uu_x + vu_y + wu_z = -p_{0\,x} + u_{yy} + u_{zz}$$

$$uv_x + vv_y + wv_z = -p_{1_y} + v_{yy} + v_{zz}$$

$$uw_x + vw_y + ww_z = -p_{1_z} + w_{yy} + w_{zz}$$

RNS 3D

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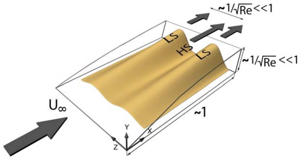

RNS 3D

$$u_x + v_y + w_z = 0$$
$$uu_x + vu_y + wu_z = -p_{0x} + u_{yy} + u_{zz}$$
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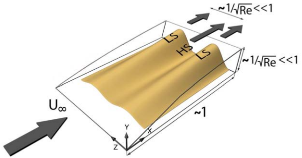

- p_1 **coupled** with u, v, w, 2nd order y-z momentum required.

RNS 3D

$$u_x + v_y + w_z = 0$$
$$uu_x + vu_y + wu_z = -p_{0x} + u_{yy} + u_{zz}$$
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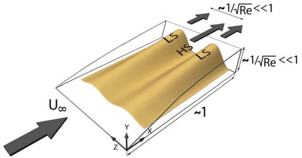

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Boundary conditions

RNS 3D

$$u_x + v_y + w_z = 0$$
$$uu_x + vu_y + wu_z = -{p_0}_x + u_{yy} + u_{zz}$$
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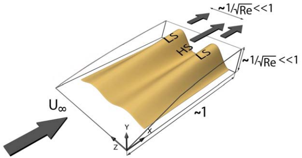

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Boundary conditions

- $u = v = w = 0$, at $y = 0$.

RNS 3D

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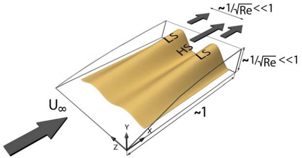

- p_1 **coupled** with u, v, w, 2nd order y-z momentum required.

Boundary conditions

- $u = v = w = 0$, at $y = 0$.
- $u = u_\infty$, $w = 0$, $v - < v >_z = 0$, as $y \to \infty$.

RNS 3D

$$u_x + v_y + w_z = 0$$

$$uu_x + vu_y + wu_z = -p_{0\,x} + u_{yy} + u_{zz}$$

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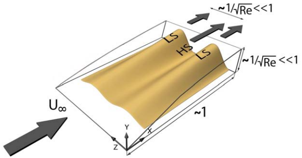

- p_1 **coupled** with u, v, w, 2nd order y-z momentum required.

Boundary conditions

- $u = v = w = 0$, at $y = 0$.
- $u = u_\infty$, $w = 0$, $v - < v >_z = 0$, as $y \to \infty$.
- Periodicty in z.

RNS 3D

$$u_x + v_y + w_z = 0$$

$$uu_x + vu_y + wu_z = -p_{0\,x} + u_{yy} + u_{zz}$$

$$uv_x + vv_y + wv_z = -p_{1\,y} + v_{yy} + v_{zz}$$

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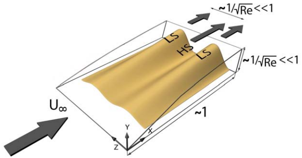

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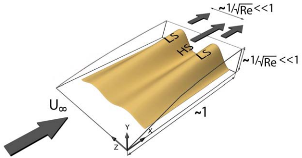

- $Re \to \infty$ Asymptotic States.

RNS 3D

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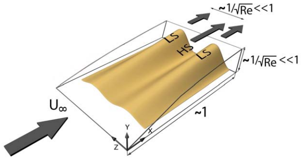

- $Re \to \infty$ Asymptotic States.
- Micro Tube flow, NL Görtler vortices with G=0 (**Hall 1998**), Streaks induced by FS perturbations (**Ricco & Luo & Wu 2011**)

RNS 3D

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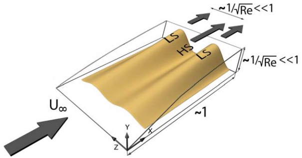
$$



- $Re \rightarrow \infty$ Asymptotic States.
- Micro Tube flow, NL Görtler vortices with G=0 (**Hall 1998**), Streaks induced by FS perturbations (**Ricco & Luo & Wu 2011**)
- Truly parabolic in x. DAE index-2 in x-direction.

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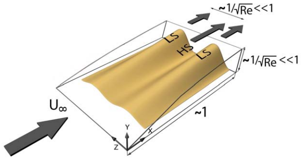

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- Truly parabolic in x. DAE index-2 in x-direction.
- 2nd order BDF x marching, 2nd order finite diff. in y and z.

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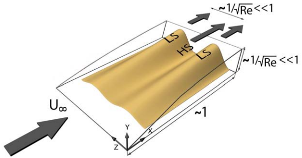

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- Improved bc at $y \to \infty$ (Higuera & Vega JFM 2009).

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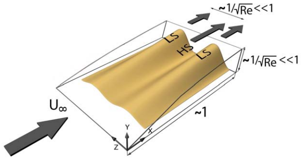
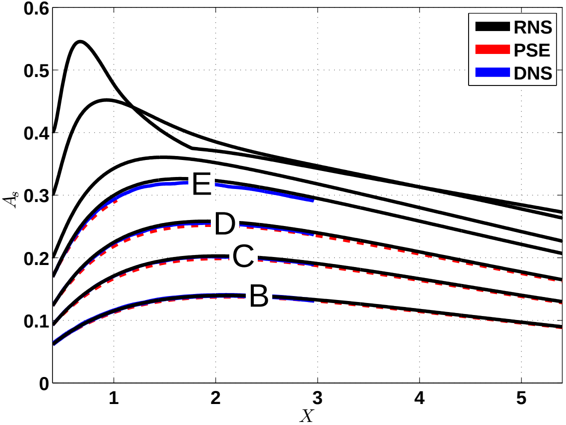


- $Re \to \infty$ Asymptotic States.
- Micro Tube flow, NL Görtler vortices with G=0 (**Hall 1998**), Streaks induced by FS perturbations (**Ricco & Luo & Wu 2011**)
- Truly parabolic in x. DAE index-2 in x-direction.
- 2nd order BDF x marching, 2nd order finite diff. in y and z.
- Improved bc at $y \to \infty$ (Higuera & Vega JFM 2009).
- Faster than DNS, more robust than NPSE.

RNS 3D results: Nonlinear Streaks



DNS: Cossu & Brandt PoF 2002, EJMB/F 2004

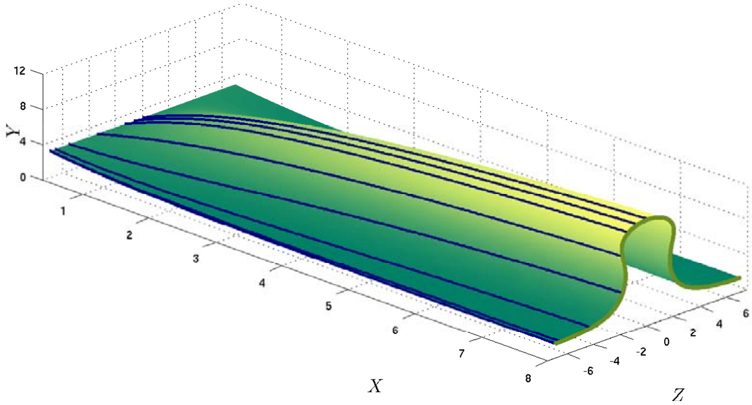
NPSE: Bagheri & Hanifi PoF 2007

RNS 3D results: Motion in transversal plane

Downstream evolution of the particles trajectories.

RNS 3D results: Motion in transversal plane

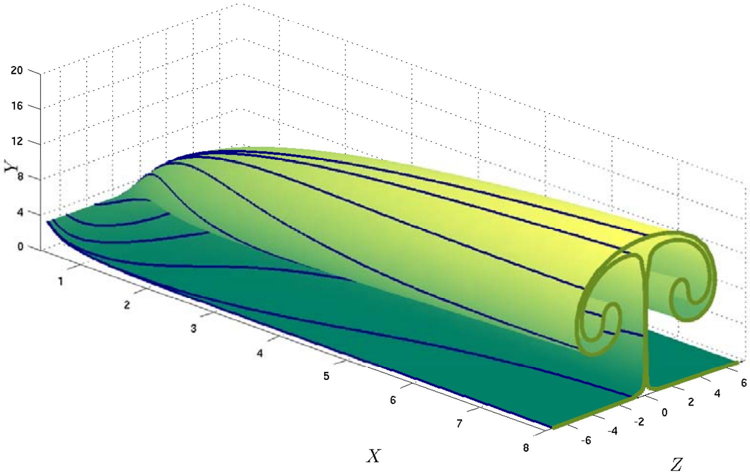
Downstream evolution of the particles trajectories.



$A_{s0} = 0.1$, departing from line $x = 0.4$, $y = 3$

RNS 3D results: Motion in transversal plane

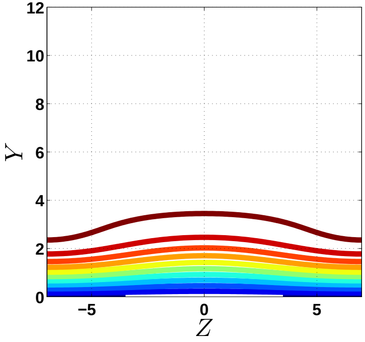
Downstream evolution of the particles trajectories.



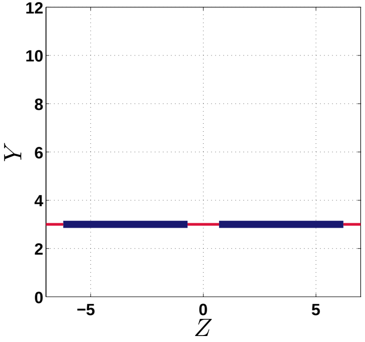
$A_{s0} = 0.4$, departing from line $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.1,\ x = 0.4}$



u constant streamwise contour lines

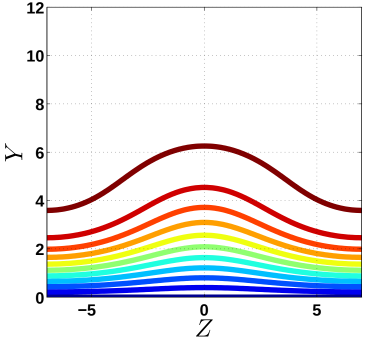


sections of particles trajectories

starting $\mathbf{x = 0.1,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.1,\ x = 1}$



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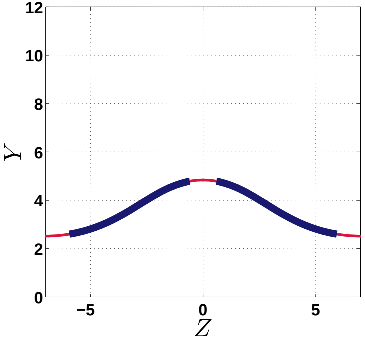
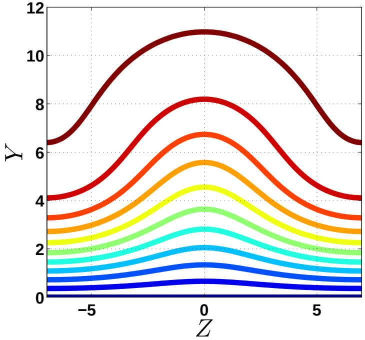


sections of particles trajectories

starting $\mathbf{x = 0.1,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.1}$, $\mathbf{x = 3}$



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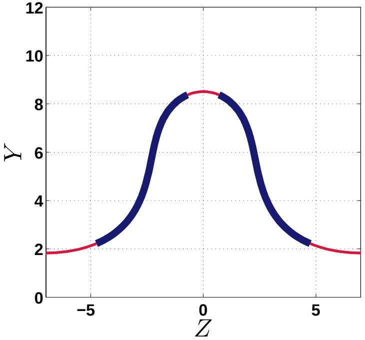
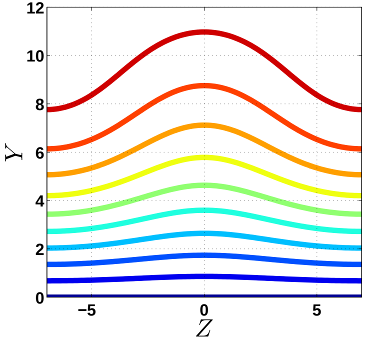


sections of particles trajectories

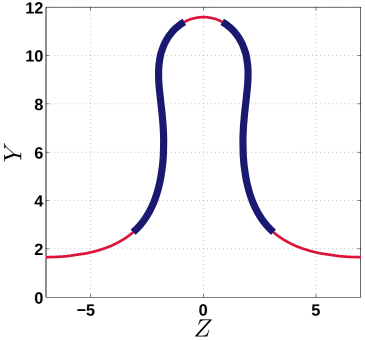
starting $\mathbf{x = 0.1}$, $\mathbf{y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.1,\ x = 7}$



u constant streamwise contour lines

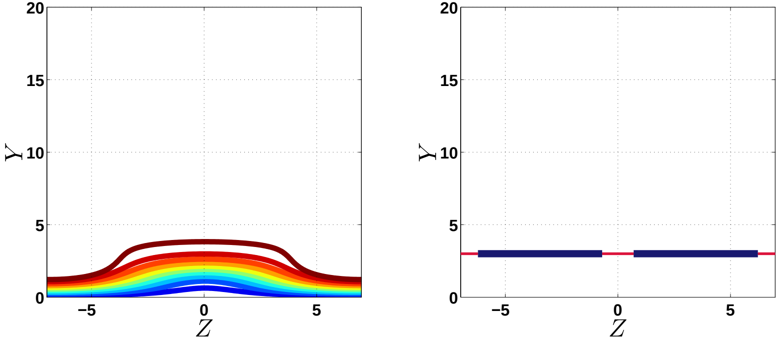


sections of particles trajectories

starting $\mathbf{x = 0.1,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.4,\ x = 0.4}$



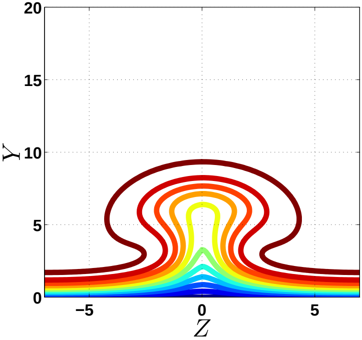
u constant streamwise contour lines

sections of particles trajectories

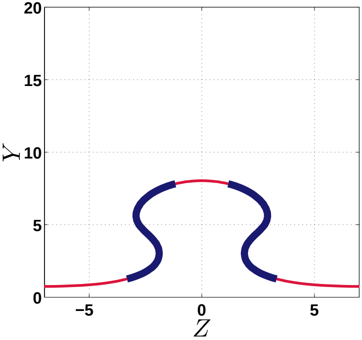
starting $\mathbf{x = 0.4,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.4,\ x = 1}$



u constant streamwise contour lines

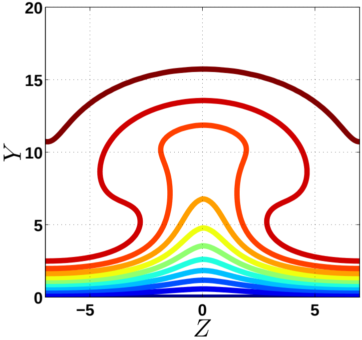


sections of particles trajectories

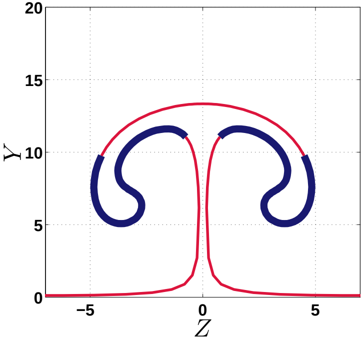
starting $\mathbf{x = 0.4,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.4,\ x = 3}$



u constant streamwise contour lines

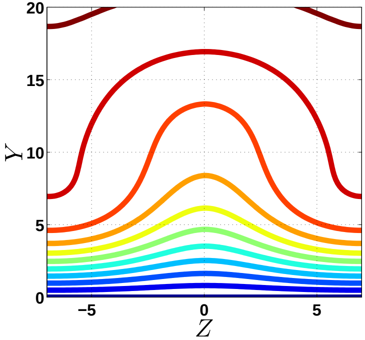


sections of particles trajectories

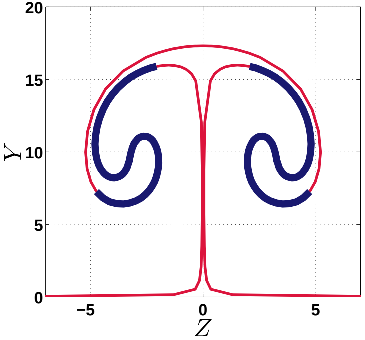
starting $\mathbf{x = 0.4,\ y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.4,\ x = 7}$



u constant streamwise contour lines

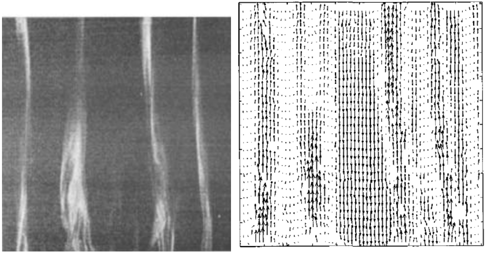


sections of particles trajectories

starting $\mathbf{x = 0.4,\ y = 3}$

RNS 3D results: Transversal structure

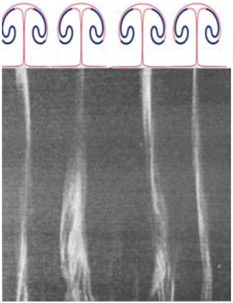
Flow visualization

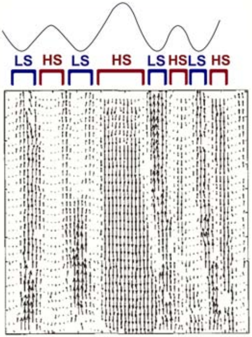


Alfredsson & Matsubara AIAA paper 2001

RNS 3D results: Transversal structure

Flow visualization

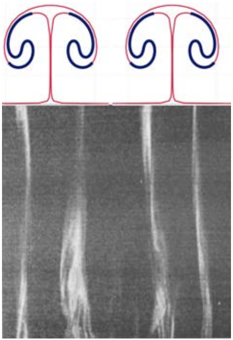


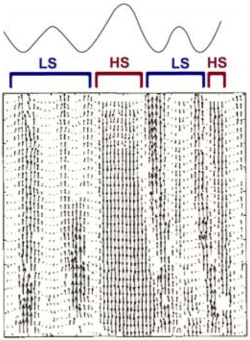


Alfredsson & Matsubara AIAA paper 2001

RNS 3D results: Transversal structure

Flow visualization





Alfredsson & Matsubara AIAA paper 2001

Two smoke traces per streak??

Discrepances in spanwise streak period

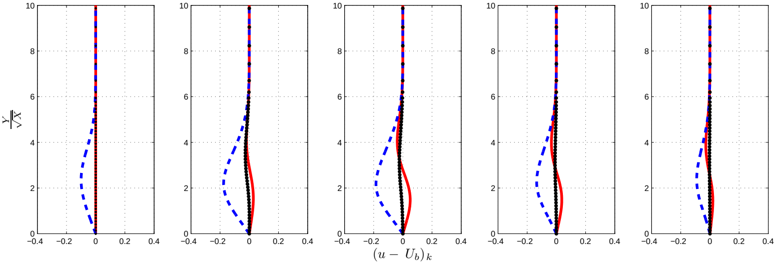
$$\lambda_{\text{expr}} \sim \lambda_{\text{LOT}}/2$$

Asai & Konishi 2007

Asai & Nishioka 1995

RNS 3D results: Effect on streamwise velocity

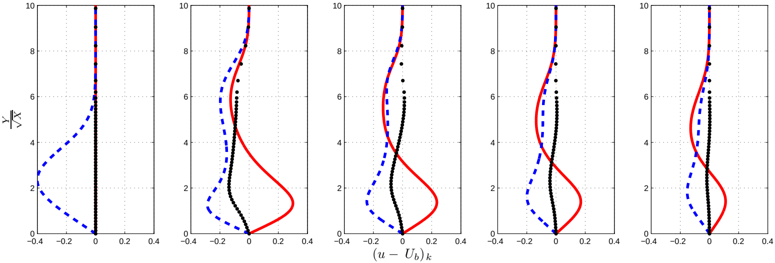
Spanwise Fourier modes of the streamwise velocity (substracting Blasius) $\mathbf{A_{s0} = 0.1}$



Mode $k = 0$, mode $k = 1$ and mode $k = 2$ at $\mathbf{x = 0.4, 1, 3, 5}$ and $\mathbf{7}$

RNS 3D results: Effect on streamwise velocity

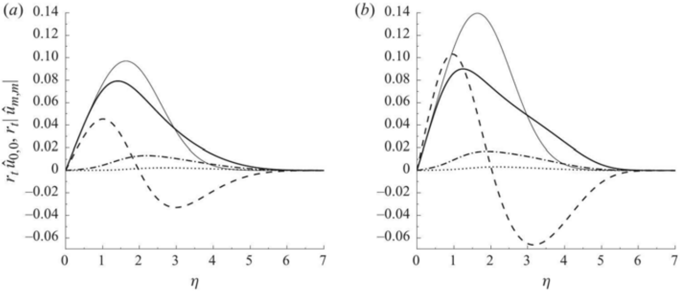
Spanwise Fourier modes of the streamwise velocity (substracting Blasius) $\mathbf{A_{s0} = 0.4}$



Mode $k = 0$, mode $k = 1$ and mode $k = 2$ at $\mathbf{x = 0.4, 1, 3, 5}$ and $\mathbf{7}$

RNS 3D results: Effect on streamwise velocity

Qualitative agreement with theorical analysis of non linear streaks



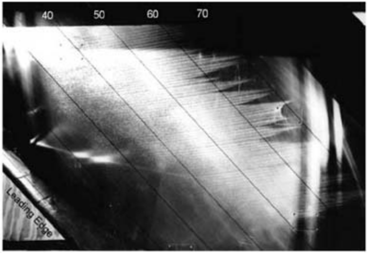
Ricco & Luo & Wu JFM 2011

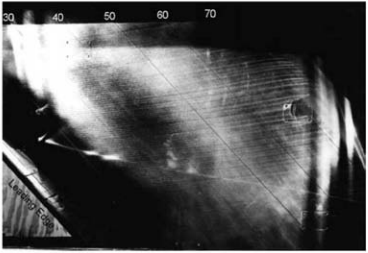
Final remarks

- RNS equations describe the downstream evolution of nonlinear streaks
    - Much less computational cost than DNS
    - More robust than PSE
- Motion in transversal plane essential to understand streak flow configuration
    - Transversal counter-rotating motion (Two smokes traces per streak??)
    - Strongly non linear effect in streamwise velocity profile
    - Not detected in linear or small amplitude streak analysis
- RNS would be probably useful to other flow configurations with two short and one long scale
    - Crossflow vortices in a swept wing
    - Edge states in plane Couette flow

X-flow Vortices

- Use RNS code to explore stability and interaction with grooves of X-flow vortices:

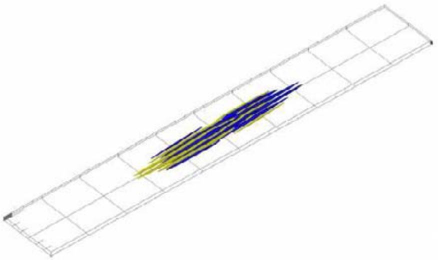


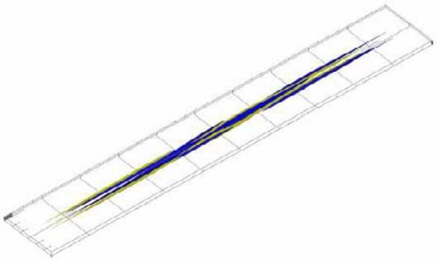


Saric & Reed AIAA 2003

Edge states

- RNS for edge states in Couette/Poiseuille flow at $Re \longrightarrow \infty$.
  Couette flow (x-localized) $Re = 375$ and $Re = 1000$.





Duguet & Schlatter & Henningson PoF 2009

Thanks for your attention!